A Simple Experiment in Relativity

Harold T. Stokes, April 2007

Albert lives on the Earth. Henry is travelling in a spaceship at a speed $v = \frac{4}{5}c$. At this speed, the Lorentz factor is $\gamma = \frac{5}{3}$. One year after Henry passes the Earth, Albert sends him a radio message: "Henry, I am sending this message one year since you passed the Earth. According to your clock, how long has it been since you passed by the Earth? Sincerely yours, Albert."

Eight years later, Albert receives a reply: "Albert, I have just now received your message. It has been three years since I passed the Earth. Sincerely yours, Henry."

Albert does some simple calculations. "It required four years for my message to reach Henry and another four years for Henry's reply to return. That means that Henry received the message five years since he passed the Earth. According to Henry's clock, it was three years since he passed the Earth. Therefore, Henry's clock is running slow by a factor of

$$(5 \text{ y}/3 \text{ y}) = \tfrac{5}{3} = \gamma$$

which agrees with the theory of special relativity."

Henry also did some simple calculations when he received the message from Albert. "Let Δt be the time since Albert sent his message. It has been three years since I passed Earth, so I have travelled a total distance equal to

$$L = (3 \text{ y})(\tfrac{4}{5}\text{c}) = \tfrac{12}{5} \text{ Ly}.$$

This must be equal to the distance $v\Delta t$ the Earth has moved since Albert sent his message plus the distance $c\Delta t$ the message has travelled, also since Albert sent his message:

$$v\Delta t + c\Delta t = L.$$

Solving for Δt, I obtain

$$\Delta t = L/(v + c) = (\tfrac{12}{5} \text{ Ly})/(\tfrac{4}{5}c + c) = \tfrac{4}{3} \text{ y}.$$

So, it has been $\frac{4}{3}$ years since Albert sent his message. This must have been $(3 \text{ y}) - \frac{4}{3} \text{ y} = \frac{5}{3}$ y after I passed the Earth. But Albert said that he sent the message one year after I passed the Earth. Therefore, Albert's clock is running slow by a factor of

$$(\tfrac{5}{3} \text{ y})/(1 \text{ y}) = \tfrac{5}{3} = \gamma$$

which agrees with the theory of special relativity."

As you can see, the correspondence proved to Albert that Henry's clock was slow and also proved to Henry that Albert's clock was slow. They are both correct.